

System of particles and rotational motion

Note : Answers given below are intended only as a reference for the questions asked in board examinations. These answers are NOT an attempt at a detailed discussion related to the question.

VSAQs

1. By spinning eggs on a table top, how will you distinguish between the hardboiled egg from a raw egg?

Spin both the eggs on a table. Stop them momentarily using your palm and quickly remove the palm. The raw egg wobbles as it has fluid inside it which does not come to rest immediately when the shell is stopped. The boiled egg remains at rest as the entire mass comes to rest when the shell is stopped.

2. Why should a helicopter necessarily have two propellers?

The change in angular momentum caused by the rotation of one propeller is balanced by the change in momentum caused by the other propeller. This ensures that the net angular momentum is zero and the helicopter remains stable against rotation. [Class notes](#)

3. Why is it easier to balance a bicycle in motion?

It is easier to balance a bicycle in motion due to gyroscopic effect by which a rotating body tends to oppose any change in its angular momentum arising out of change in the axis of rotation. [Class notes](#)

4. We cannot open or close doors by applying force at the hinges. Why?

Torque required to cause rotation is given

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$
$$\tau = r F \sin(\theta)$$

When force is applied at the hinges, $r = 0$. Therefore the torque becomes zero and the door cannot be opened or closed. [Class notes](#)

5. A child is sitting at one end of a long trolley moving with uniform speed on a smooth horizontal road. If the child gets up and runs on the trolley with speed u , what will be the state of centre of mass of trolley + child system?

Considering child + trolley as a system, as there is no external force acting on the system, the centre of mass of the system remains in equilibrium.

Force exerted by the child on the trolley (action) and the force exerted by the trolley on the child (reaction) form internal forces.

6. Newton's laws of motion are applicable to individual particles of a system. However we can also describe the motion of the system as a whole using Newton's laws. Explain.

System of particles and rotational motion

Forces acting internally on a system of particles, form action-reaction pairs. The net force acting on such a system of particles is only due to the external forces acting on it. Hence Newton's laws of motion can be applied by considering the net forces to act on the centre of mass and considering the motion of centre of mass.

$$a_{\text{cm}} = \frac{\sum F}{\sum m}$$

7. In which of the following cases the Centre of mass of a body coincides with its geometric centre?

a) The density continuously decreases from right to left.
b) The density decreases from left to right up to the centre and then Increases in the same proportion.

a) Centre of mass in this case does not coincide with geometric centre. As the density decreases from right to left, the distribution of mass is asymmetric. The right hand region contains greater mass and therefore the centre of mass lies towards the right hand side.

b) Centre of mass in this case coincides with geometric centre. As density decreases from right to centre and again increases in same proportion, the distribution of mass is symmetric. Therefore the centre of mass lies at the geometric centre.

8. Why are spokes provided in a bicycle wheel?

Spokes are provided in a bicycle wheel primarily to increase the wheel's moment of inertia resulting in higher moment of inertia without significantly adding to its overall mass. This ensures greater stability due to gyroscopic effect. Spokes also allow distribution of weight and small flexibility to the wheel. [Class notes](#)

9. Define radius of gyration. Mention the factors on which it depends.

Radius of gyration is the distance, from the axis of rotation at which a point object, of mass equal to the mass of the body, may be assumed to be located so that its moment of inertia is equal to the moment of inertia of the given body.

Moment of inertia of any body may be written in terms of radius of gyration (K) as $I = MK^2$.

10. Explain how a diver jumping into water from a height is able to make a loop in air.

When a diver is in flight, the net torque acting on him is zero his angular momentum is conserved.

$$I_1 \omega_1 = I_2 \omega_2$$

Therefore when he folds his arms and legs in his flight, his moment of inertia decreases leading to an increase in his angular velocity. [Class notes](#)

11. What happens to the duration of a day when polar ice caps melt and flow towards the equator?

System of particles and rotational motion

The net torque acting on the earth is zero its angular momentum is conserved.

$$I_1 \omega_1 = I_2 \omega_2$$

When polar ice caps melt and move towards the equator, the moment of inertia increases. This leads to a decrease in its angular velocity resulting in an increase in the duration of a day.

[Class notes](#)

- 12.** The stone of a hand flour grinder is provided with a handle near its rim. Give reason.

Torque required to rotate the grinder is given by

$$\begin{aligned}\tau &= \mathbf{r} \times \mathbf{F} \\ \tau &= r F \sin(\theta)\end{aligned}$$

If the stone of a hand flour grinder is provided with a handle near its rim, the distance (r) of the point of application of force increases and this generates the required torque with lesser force. [Class notes](#)

- 13.** It is easier to turn the steering wheel of a larger diameter than that of a small diameter. Give reason.

Torque required to rotate the steering wheel is given by

$$\begin{aligned}\tau &= \mathbf{r} \times \mathbf{F} \\ \tau &= r F \sin(\theta)\end{aligned}$$

When the diameter is large, the distance (r) of point of application of force increases and this generates the required torque with lesser force. [Class notes](#)

- 14.** A jack screw is provided with a long arm. Explain why?

Torque required to rotate the jack screw is given by

$$\begin{aligned}\tau &= \mathbf{r} \times \mathbf{F} \\ \tau &= r F \sin(\theta)\end{aligned}$$

If jack screw is provided with a long arm, the distance (r) of the point of application of force increases and this generates the required torque with lesser force. [Class notes](#)

- 15.** A person sits near the edge of a circular platform revolving with a uniform angular speed. What will be the change in the motion of the platform?

While reviewing various study materials and guide books it is found that this question is interpreted in two ways. Answers to both interpretations are given below.

There is no change in the motion in of the platform. As the person is given to be seated near the edge of a platform moving with uniform angular speed, the moment of inertia of the system (person + the platform) remains constant and hence there is no change in angular momentum.

Or

When the person sits on the platform rotating with initial uniform speed, the moment of inertia the system (person + platform) increases. This results in decrease in angular speed

System of particles and rotational motion

while angular momentum remains constant as there is no external torque acting on the system.

- 16.** Mention the factors on which moment of inertia depends.

Moment of inertia depends on

- (a) Mass of the body
- (b) Distribution of mass about the axis of rotation
- (c) Choice of axis of rotation

- 17.** Define centripetal force. Write its formula.

A force acting towards the centre of the circular path followed by a body is called centripetal force.

$$F = \frac{mv^2}{r} \quad \text{or} \quad F = mr\omega^2$$

Examples :

- (a) Electrostatic force on electron in Bohr's atom plays the role of centripetal force
- (b) Gravitational force on the earth as it revolves around the sun, plays the role of centripetal force

SAQs

- 1.** What is meant by Centre of mass of a system? Obtain expression for the Centre of mass of a system consisting of (a) two particles (b) a large number of particles. What is the physical significance of Centre of mass of a system?

Refer to class notes ([click here](#))

- 2.** Show that the velocity of Centre of mass of a moving isolated system remains constant in the absence of external force.

Acceleration of the centre of mass of a system is given by

$$a_{\text{CM}} = \frac{m_1 a_1 + m_2 a_2 + \dots}{m_1 + m_2 + \dots}$$

$$a_{\text{CM}} = \frac{F_1 + F_2 + \dots}{m_1 + m_2 + \dots}$$

$$a_{\text{CM}} = \frac{\sum F_{\text{all}}}{m_1 + m_2 + \dots}$$

When the external forces are zero, we get

$$a_{\text{CM}} = 0$$

System of particles and rotational motion

$$\frac{d\mathbf{v}_{\text{CM}}}{dt} = 0$$

$$\mathbf{v}_{\text{CM}} = \text{constant}$$

3. There is no effect on the state of centre of mass in the absence of external forces. Explain it with an example.

Refer to the answer given above

4. Give four examples of Centre of mass motion.

- Centre of mass of a bullet gun system remains stationary in the absence of external forces. Therefore when the bullet is fired from the gun, the gun recoils in a direction opposite to the direction of the bullet so that the centre of mass of the system remains unaffected.
- When a diver performs summersaults in mid air, his centre of mass follows a parabolic path as it is effected only by the gravitational force
- Centre of mass of the earth moon system undergoes epicyclic motion (describing a smaller geometric loop superimposed on a larger orbital path)
- Consider a person on trolley placed on a smooth horizontal surface. As the person moves in any direction the trolley moves in the opposite direction. This is because centre of mass of the (trolley + person) system remains at rest.

5. If net force on a system of particles is zero, what is the value of

- Acceleration of centre of mass
- Momentum of centre of mass
- velocity of centre of mass
- velocity of an individual particles of the system.

Acceleration of the centre of mass is given by

$$\mathbf{a}_{\text{CM}} = \frac{\sum \mathbf{F}_{\text{all}}}{m_1 + m_2 + \dots}$$

$$M \mathbf{a}_{\text{CM}} = \sum \mathbf{F}_{\text{all}}$$

When the external forces are zero, we get

$$M \mathbf{a}_{\text{CM}} = 0 \quad \text{---- (i)}$$

$$M \frac{d\mathbf{v}_{\text{CM}}}{dt} = 0$$

$$M \mathbf{v}_{\text{CM}} = \text{constant} \quad \text{---- (ii)}$$

$$\mathbf{v}_{\text{CM}} = \text{constant} \quad \text{---- (iii)}$$

$$\mathbf{v}_{\text{CM}} = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 + \dots}{m_1 + m_2 + \dots} \quad \text{---- (iv)}$$

System of particles and rotational motion

- (a) From equation (i), acceleration of centre of mass is zero
- (b) From equation (ii), momentum of centre of mass is constant
- (c) From equation (iii), velocity of centre of mass is constant
- (d) From equation (iv), velocity of individual particles may or may not be zero

6. Distinguish between Centre of mass and Centre of gravity.

Centre of mass	Centre of gravity
Centre of mass is the point at which the entire mass of the body is assumed to be concentrated	Centre of gravity is the point at which the entire gravitational force (weight) of the body is assumed to act
$\mathbf{r}_{CM} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2 + \dots}{m_1 + m_2 + \dots}$	$\mathbf{r}_{CG} = \frac{m_1 g_1 \mathbf{r}_1 + m_2 g_2 \mathbf{r}_2 + \dots}{m_1 g_1 + m_2 g_2 + \dots}$
Centre of mass is independent of the acceleration due to gravity	Centre of gravity depends on the acceleration due to gravity over the dimensions of the body

For small objects centre of mass coincides with centre of gravity. For large objects centre of mass does not coincide with centre of gravity.

7. Show that a system of particles moving under the influence of an external force, moves as if the force is applied at its centre of mass.

Acceleration of the centre of mass of a system of particles is given by

$$\mathbf{a}_{CM} = \frac{m_1 \mathbf{a}_1 + m_2 \mathbf{a}_2 + \dots}{m_1 + m_2 + \dots}$$

$$\mathbf{a}_{CM} = \frac{\mathbf{F}_1 + \mathbf{F}_2 + \dots}{m_1 + m_2 + \dots}$$

$$\mathbf{a}_{CM} = \frac{\sum \mathbf{F}_{all}}{m_1 + m_2 + \dots}$$

When the external forces are zero, we get

$$(m_1 + m_2 + \dots) \mathbf{a}_{CM} = \sum \mathbf{F}_{all}$$

As the internal forces (action – reaction pairs) get cancelled we get

$$\sum \mathbf{F}_{ext} = (m_1 + m_2 + \dots) \mathbf{a}_{CM}$$

Therefore system of particles moving under the influence of an external force, moves as if the force is applied at its centre of mass

System of particles and rotational motion

8. Explain about the centre of mass of earth-moon system and its rotation around the sun.

Centre of mass of the earth – moon system lies closer to the centre of the earth due to its larger mass. The speed of earth w.r.t. the sun is also much greater than the speed of the moon w.r.t. the earth. Due to these conditions, centre of mass of the earth – moon system may be assumed to move in an elliptical path. The small variations from the elliptical path observed are due to distance between the earth and the moon as seen from sun.

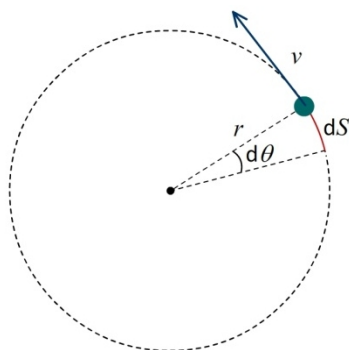
External resource : [Click here for a excellent simulation !!](#)

9. Define angular velocity (ω). Derive $v = r \omega$.

The rate of change of angular displacement w.r.t. time is called angular velocity.

$$\omega = \frac{d\theta}{dt}$$

Consider a body moving along a circular path of radius r with uniform angular velocity ω . Consider a small displacement (dS) of the body along its path. The angle subtended by it at the centre is $d\theta$.



Using the relation arc = radius $\times$ angle we get

$$dS = r d\theta$$

differentiating this with respect to time we get

$$\frac{dS}{dt} = r \frac{d\theta}{dt}$$

By definition, dS/dt is velocity (v) and $d\theta/dt$ is angular velocity (ω) therefore

$$v = r \omega$$

10. Define angular acceleration and torque. Establish the relation between angular acceleration and torque.

[Refer to class notes](#)

11. Write the equations of motion for a particle rotating about a fixed axis.

[Refer to class notes](#)

System of particles and rotational motion

- 12.** Write the expressions for (a) Torque (b) Angular momentum (c) Rotational power (d) Rotational kinetic energy.

(a) Torque $\tau = \mathbf{r} \times \mathbf{F}$ or $\tau = I\alpha$ or $\tau = \frac{dL}{dt}$

(b) Angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ or $L = I\omega$

(c) Rotational power $P = \frac{dKE_R}{dt}$ or $P = \tau\omega$

(d) Rotational kinetic energy $KE_R = \frac{1}{2} I\omega^2$

- 13.** Write the expression for moment of Inertia of (a) Solid sphere of radius R about its diameter (b) Circular ring of radius R through its centre and perpendicular to its plane (c) Disc of radius R through its centre and perpendicular to its plane (d) Solid cylinder of length l and radius R through its centre and perpendicular to its length.

(a) Solid sphere of radius R about its diameter $KE_R = \frac{1}{2} I\omega^2$

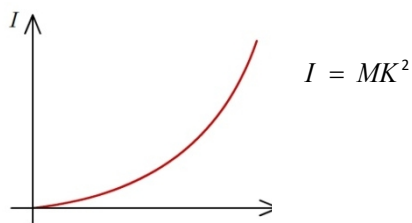
(b) Circular ring of radius R through its centre and perpendicular to its plane

(c) Disc of radius R through its centre and perpendicular to its plane

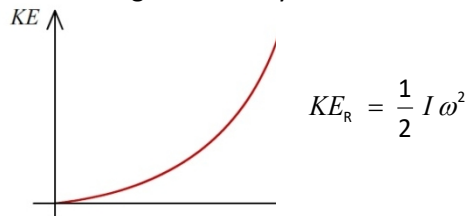
(d) Solid cylinder of length l and radius R through its centre and perpendicular to its length

- 14.** Draw the Graph between

- (a) Moment of Inertia and radius of gyration.

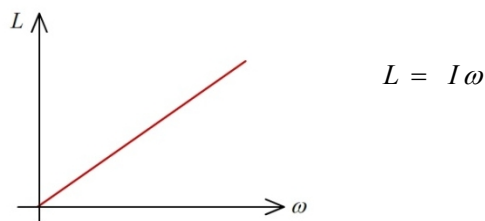


- (b) Rotational K.E and angular velocity.

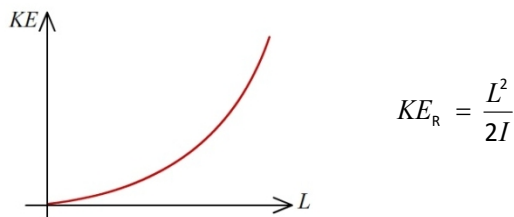


- (c) Angular momentum and angular velocity

System of particles and rotational motion

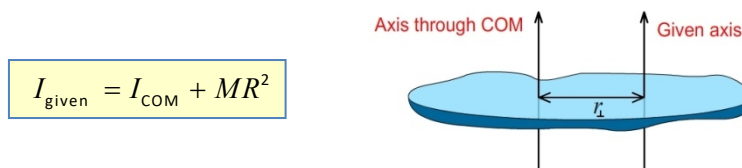


(d) Angular momentum and Kinetic energy of rotation.



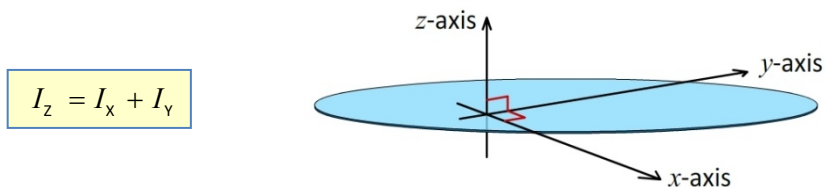
15. State Parallel axis theorem and give expression.

Moment of inertia of a body about a given axis is equal to the sum of its moment of inertia about an axis passing through its centre of mass and the product of mass of the body and square of the perpendicular distance between the two axes.



16. State Perpendicular axis theorem and give expression.

Moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two mutually perpendicular axes in its plane with the perpendicular axis passing through their point of intersection.



LAQs

1. – coming soon --
- 2.
- 3.

System of particles and rotational motion

4.

5.